7.4 RESOLUTION PERFORMANCE ASSESSMENT FOR QUADRATIC TFDs [0]

7.4.1 Selecting and Comparing TFDs

Quadratic time-frequency distributions (TFDs) are effective tools for extracting information from a non-stationary signal, such as the number of components, their durations and bandwidths, components' relative amplitudes and instantaneous frequency (IF) laws (see Chapters 1 and 2). The performance of TFDs depends on the type of signal (see Chapter 3) [1, 2]. For example, in the case of a monocomponent linear FM signal, the Wigner-Ville distribution is known to be optimal in the sense that it achieves the best energy concentration around the signal IF law (see Article 2.1 for more details) [1].

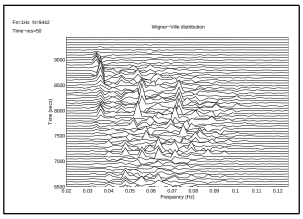
In applications involving multicomponent signals, choosing the right TFD to analyze the signals is an immediate critical task for the signal analyst. How best to make this assessment, using current knowledge, is the subject of this article.

Let us, for example, consider a multicomponent whale signal, represented in the time-frequency domain using the Wigner-Ville distribution, the spectrogram, the Choi-Williams distribution, the Born-Jordan distribution, the Zhao-Atlas-Marks (ZAM) distribution, and the recently introduced B-distribution [3] (see Fig. 7.4.1).

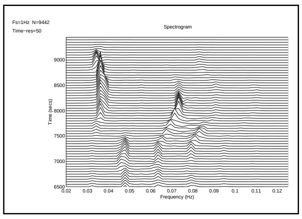
To determine which of the TFDs in Fig. 7.4.1 "best" represents this whale signal (i.e. which one gives the best components' energy concentration and best interference terms suppression, and allows the best estimation of the components' IF laws) one could *visually* compare the six plots and choose the most appealing. The spectrogram and the B-distribution, being almost free from the cross-terms, seem to perform best.

The performance comparison based on the visual inspection of the plots becomes more difficult and unreliable, however, when the signal components are closely-spaced in the time-frequency plane. To objectively compare the plots in Fig. 7.4.1 requires to use a quantitative performance measure for TFDs. There have been several attempts to define objective measures of "complexity" for TFDs (see Section 7.3.1). One of these measures, the Rényi entropy given in [4], has been used by several authors in preference to e.g. the bandwidth–duration product given in [1]. The performance measure described in this article, unlike the Rényi entropy, is a local measure of the TFD resolution performance, and is thus more suited to the selection problem illustrated by Fig. 7.4.1. This measure takes into account the characteristics of TFDs that influence their resolution, such as energy concentration, components separation, and interference terms minimization. Methodologies for choosing a TFD which best suits a given signal can then be developed by optimizing the resolution performance of considered TFDs and modifying their parameters to better match application-specific requirements.

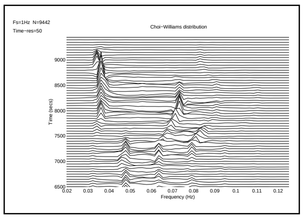
[0] Authors: **Boualem Boashash** and **Victor Sucic**, Signal Processing Research Centre, Queensland University of Technology, GPO Box 2434, Brisbane, Q 4001, Australia (b.boashash@qut.edu.au, v.sucic@qut.edu.au). Reviewers: W. J. Williams and LJ. Stanković.



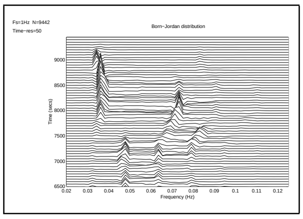
(a) Wigner–Ville



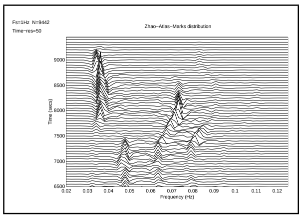
(b) Spectrogram (Hanning, $L = 511$)



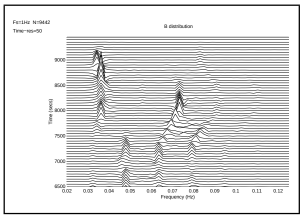
(c) Choi-Williams ($\sigma = 10$)



(d) Born–Jordan



(e) ZAM ($a = 2$)



(f) B ($\beta = 0.01$)

Fig. 7.4.1: TFDs of a multicomponent whale signal.

7.4.2 Performance Criteria for TFDs

In the case of *monocomponent* FM signals, the best TFD is that which maximizes energy concentration about the signal instantaneous frequency. This is achieved by minimizing component sidelobe amplitude A_s relative to mainlobe amplitude A_m, and mainlobe bandwidth B relative to central frequency f (see Fig. 7.4.2).

The instantaneous concentration performance of a TFD may thus be quantified by the measure p expressed as:

$$p(t) = \left| \frac{A_s(t)}{A_m(t)} \right| \frac{B(t)}{f(t)} \tag{7.4.1}$$

A good performance is characterized by a small value of the measure p. For example, for the Wigner-Ville distribution of a linear FM signal with infinite duration, the bandwidth B and the sidelobe amplitude A_s are zero [1], and we obtain $p = 0$.

For *multicomponent* FM signals, the performance of a TFD can be *quantitatively* assessed in terms of:

- the energy concentration of the TFD about the respective instantaneous frequency of each component, as expressed by Eq. (7.4.1), and
- the components resolution, as measured by the frequency separation of the components' mainlobes, including the effect of cross-terms.

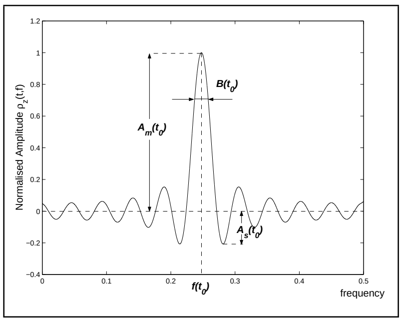


Fig. 7.4.2: Slice of a TFD of a monocomponent signal at time $t = t_0$. The dominant peak is the component, while the other peaks are the sidelobes. For clarity of presentation, we limit ourselves to measuring the mainlobe bandwidth at 0.71 of the component normalized amplitude A_m.

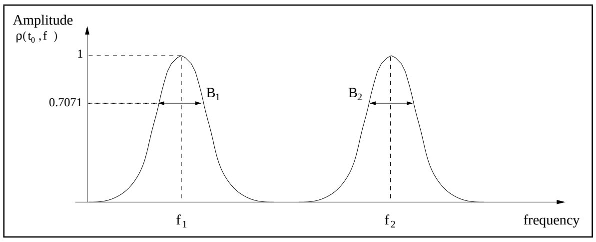


Fig. 7.4.3: Diagram illustrating the resolution of a two-component signal in the absence of cross-terms. The lobes are clearly distinguished from each other; the components are said to be resolved.

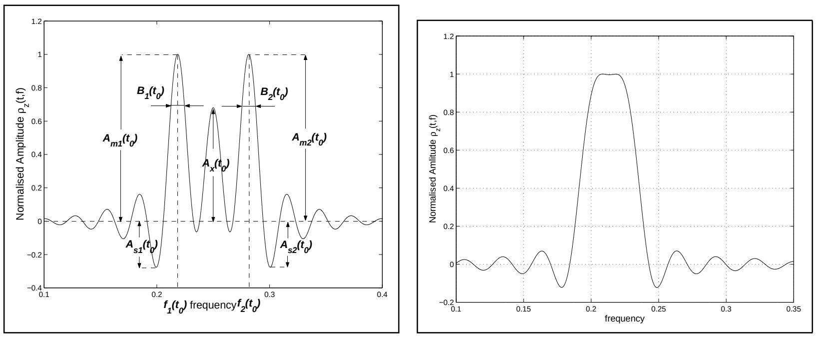
For stationary signals, the frequency resolution in a power spectral density estimate of a signal composed of two single tones, f_1 and f_2 (see Fig. 7.4.3), may be defined as the minimum difference $f_2 - f_1$ for which the following inequality holds:

$$f_1 + B_1/2 < f_2 - B_2/2\,, \qquad f_1 < f_2 \tag{7.4.2}$$

where B_1 and B_2 are the respective bandwidths of the first and the second sinusoid.

In the case of non-stationary signals, for a TFD $\rho_z(t, f)$ of a two-component signal, the above definition of resolution is valid for every time slice of a cross-term-free TFD, such as the spectrogram. However, for TFDs exhibiting cross-terms, we need to take into account the effect of cross-terms on resolution.

A slice of a typical quadratic TFD, with components clearly resolved, is shown in Fig. 7.4.4(a), where $B_1(t_0)$, $f_1(t_0)$, $A_{s_1}(t_0)$ and $A_{m_1}(t_0)$ represent respectively the instantaneous bandwidth, the IF, the sidelobe amplitude and the mainlobe amplitude of the first component at time $t = t_0$. Similarly, $B_2(t_0)$, $f_2(t_0)$, $A_{s_2}(t_0)$ and $A_{m_2}(t_0)$ represent the instantaneous bandwidth, the IF, the sidelobe amplitude and the mainlobe amplitude of the second component at the same time t_0. The amplitude $A_x(t_0)$ is that of the cross-term. An example of a quadratic TFD with non-resolved components is shown in Fig. 7.4.4(b).



(a) The two dominant peaks are the (resolved) signal components, the middle peak is the cross-term, and the other peaks are the sidelobes

(b) The two components and the cross-term have merged into a single peak; we say that the components are *not* resolved

Fig. 7.4.4: Slice of a TFD of a two-component signal taken at time $t = t_0$.

7.4.3 Resolution Performance Measure for TFDs

Eq. (7.4.2) and Fig. 7.4.4(a) suggest that the resolution performance of a TFD for a neighboring pair of components in a multicomponent signal may be defined by the minimum difference $D(t) = f_2(t) - f_1(t)$ for which we still have a positive separation measure $S(t)$ between the components' mainlobes centered about their respective IFs, $f_1(t)$ and $f_2(t)$. For best resolution performance of TFDs, $S(t)$ should be as close as possible to the true difference between the actual IFs.

The components' separation measure $S(t)$ is expressed as [5]:

$$S(t) = \left(f_2(t) - \frac{B_2(t)}{2} \right) - \left(f_1(t) + \frac{B_1(t)}{2} \right) \tag{7.4.3}$$

The resolution also depends on the following set of variables, all of which should be as small as possible:

(a) the normalized *instantaneous bandwidth* of the signal component $B_k(t)/f_k(t)$, $k = 1, 2$, which is accounted for in $S(t)$ (Eq. (7.4.3)),

(b) the ratio of the *sidelobe amplitude* $|A_{s_k}(t)|$ to the *mainlobe amplitude* $|A_{m_k}(t)|$, $k = 1, 2$, of the components, and

(c) the ratio of the *cross-term amplitude* $|A_x(t)|$ to the *mainlobe amplitudes* of the signal components $|A_{m_k}(t)|$, $k = 1, 2$.

It follows that the best TFD for multicomponent signals analysis is the one that concurrently *minimizes* the positive quantities (a), (b), (c), and *maximizes* $S(t)$.

Hence, by combining the above variables, expressions for a measure $P(t)$ of the resolution performance of a given TFD can be defined. Two have been proposed

in [5], among these a normalized performance measure expressed as:

$$P(t) = 1 - \frac{1}{3}\left\{\left|\frac{A_s(t)}{A_m(t)}\right| + \frac{1}{2}\left|\frac{A_x(t)}{A_m(t)}\right| + \left(1 - \frac{S(t)}{D(t)}\right)\right\} \tag{7.4.4}$$

where, for a pair of signal components, $A_m(t)$ and $A_s(t)$ are respectively the average amplitudes of the components' mainlobes and sidelobes, $A_x(t)$ is the cross-term amplitude, $S(t)$, defined by Eq. (7.4.3), is a measure of the components' separation in frequency, and $D(t) = f_2(t) - f_1(t)$ is the difference between the components' actual IFs. The algorithm presented in [6] describes how the parameters in Eq. (7.4.4) are measured in practice.

The measure $P(t)$ is close to 1 for well-performing TFDs and 0 for poorly-performing ones. Therefore, when designing a TFD we want to maximize $P(t)$ in order to reduce the cross-terms, while preserving the components' resolution.

In some applications involving real-life signals, we may need to better discriminate between different TFDs resolution performances in a given set of K TFDs. In this case, a suitable alternative to $P(t)$ that was proposed in [7] could be used. It is expressed as:

$$M_j(t) = 1 - \frac{1}{3}\left\{\frac{\left|\frac{A_{s_j}(t)}{A_{m_j}(t)}\right|}{\max\limits_{1\le k\le K}\left(\left|\frac{A_{s_k}(t)}{A_{m_k}(t)}\right|\right)} + \frac{\left|\frac{A_{x_j}(t)}{A_{m_j}(t)}\right|}{\max\limits_{1\le k\le K}\left(\left|\frac{A_{x_k}(t)}{A_{m_k}(t)}\right|\right)} + \frac{\frac{B_j(t)}{D_j(t)}}{\max\limits_{1\le k\le K}\left(\frac{B_k(t)}{D_k(t)}\right)}\right\} \tag{7.4.5}$$

where $M_j(t)$ $(1 \le j \le K)$ is the resolution performance measure of the j–th TFD, and B is the average instantaneous bandwidth of the components mainlobes. The measure $M(t)$ is used in Section 7.4.5 to compare the performances of quadratic TFDs of a real-life signal, as it discriminates better than the measure $P(t)$ for real-life signals [8].

7.4.4 Application to the Selection of the Optimal TFD for a Given Multicomponent Signal

A methodology for selecting the optimal TFD for resolving closely-spaced components in a multicomponent signal involves then the following steps:

1. Define a set of comparison criteria describing the information sought from TFDs (Section 7.4.2).

2. Objectively measure the resolution performance of TFDs based on these criteria (use the measure P defined by Eq. (7.4.4)).

3. Optimize each TFD to match the criteria as close as possible [5,6]: Select as the optimal TFD kernel parameter value the one which maximizes the overall performance measure P_{overall}, taken to be the mean of the instantaneous measures P in a time interval of interest.

Table 7.4.1: Optimization results for the TFDs of signal $s(t)$ defined by Eq. (7.4.6).

TFD	*Optimal value of the kernel parameter*	P_{overall}
Spectrogram	Bartlett window, length 31	0.86
Wigner-Ville	N/A in this case	0.62
Choi-Williams	$\sigma = 1$	0.82
Born-Jordan	N/A in this case	0.81
Zhao-Atlas-Marks (ZAM)	$a = 2$	0.67
Modified B	$\beta = 0.04$	0.88

4. Quantitatively compare TFDs and select the best one: An optimized TFD which has the largest value of P_{overall} is selected as *best* for representing the given signal in the joint time-frequency domain.

Example: We define the following two-component signal in noise:

$$\begin{aligned} s(t) &= s_1(t) + s_2(t) + n(t) \\ &= \cos\left(2\pi\left(0.1\,t + \tfrac{\alpha}{2}\,t^2\right)\right) + \cos\left(2\pi\left(0.2\,t + \tfrac{\alpha}{2}\,t^2\right)\right) + n(t) \end{aligned} \qquad (7.4.6)$$

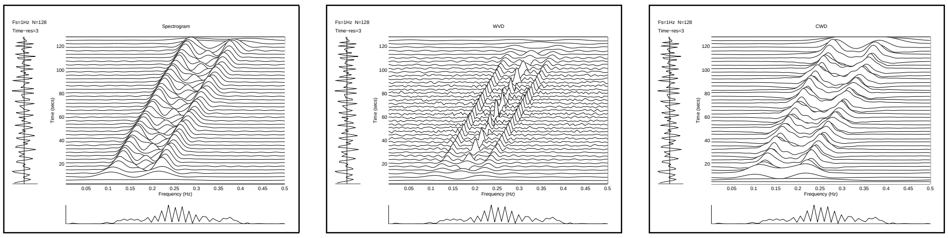
where $\alpha = 0.0016$ is the component bandwidth-duration ratio (duration $T = 128$), and $n(t)$ is additive white Gaussian noise with signal-to-noise ratio SNR $= 10$ dB. The sampling frequency is $f_s = 1$ Hz.

The signal $s(t)$ is analyzed in the time-frequency domain using the following TFDs: the spectrogram, the Wigner-Ville distribution, the Choi-Williams distribution, the Born-Jordan distribution, the Zhao-Atlas-Marks (ZAM) distribution, and the Modified B-distribution [9].

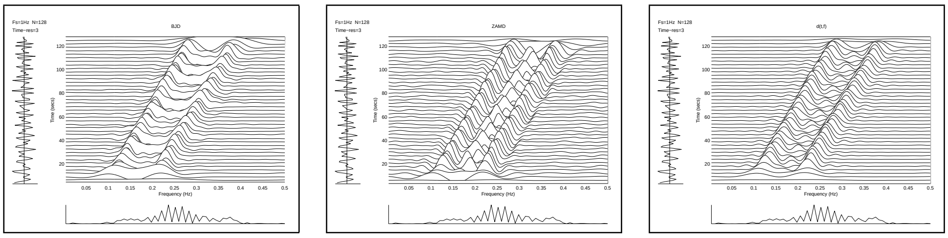
To find the optimal TFD for resolving the two components of $s(t)$, we first find the optimal values of the TFDs kernel parameters, as described in the above methodology. The Wigner-Ville distribution and the Born-Jordan distribution have no "smoothing" parameters, hence do not need optimizing. The optimized TFD with the largest P_{overall} among the considered TFDs is then selected as optimal for representing $s(t)$. Table 7.4.1 lists the results of the optimization process, and it shows that the signal optimal TFD is the Modified B-distribution with the parameter $\beta = 0.04$. All optimized TFDs are plotted in Fig. 7.4.5. From the signal optimal TFD important signal parameters can be measured (see Table 7.4.2). In addition, by optimizing components' concentration and resolution, more accurate components IF laws' estimates are obtained from the peaks of the optimal TFD's dominant ridges in the time-frequency plane [1] (see Fig. 7.4.6).

7.4.5 Use of the Performance Measure in Real-Life Situations

The methodology defined in this section enables to select a real-life signal best-performing TFD in an objective, automatic way. Its use should make time-frequency techniques more applicable in practice (e.g. machine condition monitoring described in Articles 15.2 and 15.6, or other applications presented in Part V).



(a) Spectrogram (Bartlett, $L = 31$) (b) Wigner–Ville (c) Choi-Williams ($\sigma = 1$)



(d) Born–Jordan (e) ZAM ($a = 2$) (f) Modified B ($\beta = 0.04$)

Fig. 7.4.5: Optimized TFDs of signal $s(t)$ defined by Eq. (7.4.6).

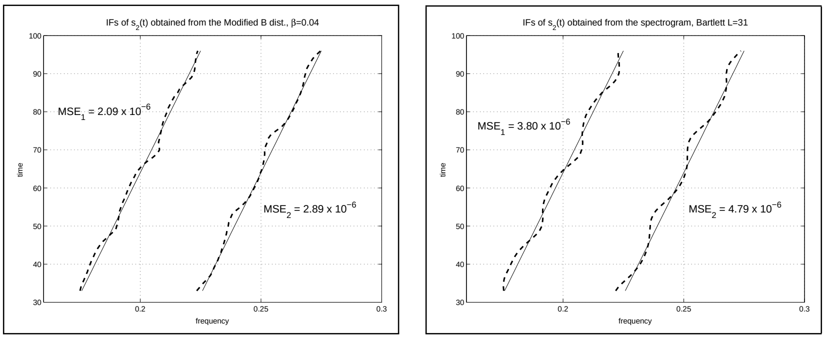
Table 7.4.2: Parameters of $s_1(t)$ and $s_2(t)$ (Eq. (7.4.6)) measured from the signal $s(t)$ optimal TFD (Modified B-distribution, $\beta = 0.04$). The values shown are the averages over $t \in [32, 96]$.

Parameter	*Component* $s_1(t)$	*Component* $s_2(t)$
Instantaneous bandwidth B	0.0194	0.0195
Mainlobe amplitude $\|A_m\|$	1.0002	0.9574
Sidelobe amplitude $\|A_s\|$	0.0900	0.0858
Cross-term amplitude $\|A_x\|$	0.1503	

The methodology consists of the following steps:

1. Represent the signal in the time-frequency domain with a quadratic TFD, i.e. a smoothed Wigner-Ville distribution (see Article 3.2). Following the approach described in Article 5.7, we smooth the WVD in both time t and lag τ with the Hanning window of length equal to a quarter of the signal duration. This time-frequency smoothing is intended to suppress the WVD inner and outer artifacts, while preserving components time-frequency features (see Article 4.2 for more details).

2. For the different time instants of the smoothed WVD, select the two closest dominant peaks in the frequency direction. To achieve the best resolution of



(a) Modified B ($\beta = 0.04$) (b) Spectrogram (Bartlett, $L = 31$)

Fig. 7.4.6: Comparison of the measured (dashed) and true (solid) IF laws of the component $s_1(t)$ (left) and $s_2(t)$ (right) of the signal $s(t)$ defined by Eq. (7.4.6). The mean-square-errors (MSEs) of the IF estimates obtained from the peaks of the signal optimized Modified B-distribution (best-performing TFD) are given in (a), and those obtained from the peaks of the signal optimized spectrogram (second best TFD) in (b).

the signal components, the best resolution of the two closest components at an observed time instant is sufficient [10]. Note that if a signal is monocomponent or no components exist at a particular time, this time instant is not considered.

3. For the selected pairs of components, optimize different TFDs using the resolution performance measure M defined by Eq. (7.4.5). The measure M is used over P since it is a better discriminator of real-life signals TFDs resolution performances [11]. The kernel parameter value, which from a set of different values considered, maximizes the overall performance measure (the mean of M over the observed times) is selected as the kernel parameter optimal value.

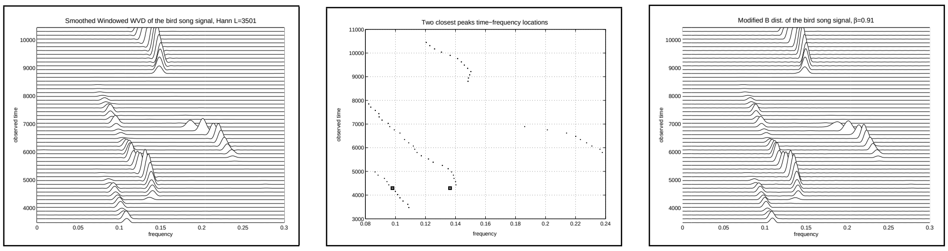
4. Calculate the measure M of the optimized TFDs for each of the selected pairs of signal components. The TFD which maximizes the average (over time) M is selected as the signal *best-performing* TFD among the considered TFDs.

Example: To illustrate how to use this methodology in practice, let us find the best-performing TFD for the Noisy Miner (*Manorina melanocephala*) song signal. The same TFDs we considered in the synthetic signal optimal TFD selection example will be considered in this real-life signal example.

We start by representing the signal in the time-frequency domain with the Wigner-Ville distribution smoothed in time and lag with the Hanning windows of length $L = 3501$ (Fig. 7.4.7(a)).

For each time instant of the smoothed WVD we then identify the pair of closest components. From Fig. 7.4.7(b) we can see that different components form such pairs at different times.

Next, as described in steps 3 and 4 of the above-defined methodology, the six considered TFDs are first optimized, after which their resolution performances are



(a) Smoothed WVD (Hanning, $L = 3501$)

(b) Pairs of closest components

(c) Modified B ($\beta = 0.91$)

Fig. 7.4.7: Optimization of the bird song signal TFDs. The signal Smoothed WVD is shown in (a), and the pairs of its closest components in (b), with the overall closest pair (at time $t = 4295$) marked by the squares. The signal optimized Modified B-distribution is shown in (c).

Table 7.4.3: Optimization and comparison results for the TFDs of the Noisy Miner song signal. The values of M_{overall} indicate that the spectrogram performs better than most traditional TFDs in this case. Only the Modified B-distribution performs better than the spectrogram and all others.

TFD	*Optimal value of the kernel parameter*	M_{overall}
Spectrogram	Bartlett window, length 3501	0.90
Wigner-Ville	N/A in this case	0.50
Choi-Williams	$\sigma = 0.004$	0.74
Born-Jordan	N/A in this case	0.65
Zhao-Atlas-Marks (ZAM)	$a = 2$	0.63
Modified B	$\beta = 0.91$	0.93

evaluated using the measure M. Table 7.4.3 shows the signal TFDs kernel parameters optimization and the TFDs resolution performance results. The Modified B-distribution for $\beta = 0.91$, plotted in Fig. 7.4.7(c), is found to have the largest value of M_{overall} (the mean of M over the time instants). Therefore, we select this TFD as best to represent the Noisy Miner song signal in the time-frequency plane.

7.4.6 Summary and Conclusions

This article defines a measure for assessing the resolution performance of quadratic TFDs in separating closely-spaced components in the time-frequency domain. The measure takes into account key attributes of TFDs, such as components' mainlobes and sidelobes, and cross-terms. The introduction of this measure allows to quantify the quality of TFDs instead of relying solely on visual inspection of plots. The resolution performance measure also allows for selecting the optimal TFD in a given practical application, and improving methodologies for designing high resolution quadratic TFDs, such as the Modified B-distribution.

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